1. Please answer the following questions.
(a) If $A$ has $n$ elements and $B$ has $m$ elements, how many different relations are there form $A$ to $A \times B$? Please explain your answer in brief. (5%)
(b) If $|A| = 30$ and the equivalence relation $R$ on $A$ partitions $A$ into three equivalence classes $A_1$, $A_2$, and $A_3$, where $|A_1| = 12$, $|A_2| = 8$, and $|A_3| = 10$, what is $|R|$? Please explain your answer in brief. (10%)

2. Let $R$ be a relation on a set $A$.
(a) What is the transitive closure of a relation $R$? (5%)
(b) Let $A = \{a, b, c, d, e, f\}$ and $R = \{(a, a), (a, d), (b, b), (b, d), (c, d), (c, e), (d, a), (e, b), (e, e)\}$ be a relation on $A$. Please find the matrix of the transitive closure of the relation $R$. (10%)

3. The Fibonacci number is defined recursively by
   
   (1) $F_0 = 1$, $F_1 = 1$; and
   
   (2) $F_n = F_{n-1} + F_{n-2}$ for $n \in \mathbb{Z}^+$ with $n \geq 2$.

   Please prove $F_n \leq (5/3)^n$ for all $n \{0, 1, 2, \ldots\}$ (10%)

4. Please answer the following questions.
(a) Given $G = (V, E)$ is a connected planar simple graph with $|E| > 1$. Prove that $|E| \leq 3|V| - 6$. (10%)
(b) Prove that $K_5$ is not a planar graph. (10%)

5. There are $n$ straight lines on a plane, each pair of lines must intersect at a point and no three lines meet at a common point. How many regions that was divided by such $n$ lines? Please give the recurrence relation and solve it. (20%)

6. Select three distinct numbers from 101 to 190. How many choice of them satisfy the following condition: the sum of these three numbers will be divided by 3? (10%)

7. How many nonnegative integer solutions are there of $x_1 + x_2 + x_3 + x_4 = 34$, where $x_1 \geq 2$, $x_1 \geq 0$, $x_3 > 1$, and $x_4 \geq 5$. (10%)