1. (15%) Evaluate 
   \[ \int_0^5 \sqrt{25 - 9x^2} \, dx \] \hspace{1cm} \( \int x \sqrt{x - 1} \, dx \) \hspace{1cm} \( \lim_{x \to 0} (1 + x)^{1/2} \).

2. (5%) Find \( \frac{d^2y}{dx^2} \) of \( x^2 - y^2 = 16 \).

3. (10%) Find the minimum value of \( f(x, y, z) = 2x + y + 2z \) subject to the constraint \( x^2 + y^2 + z^2 = 4 \).

4. (10%) Is the matrix
   \[
   \begin{bmatrix}
   1 & 1 & 1 & 1 \\
   1 & 2 & -1 & 2 \\
   1 & -1 & 2 & 1 \\
   1 & 3 & 3 & 2
   \end{bmatrix}
   \]
   singular or nonsingular? If it is nonsingular, find its inverse.

5. (10%) Which of the following sets of vectors form a basis for \( \mathbb{R}^3 \)?
   (a) \{ (1, 1, 1), (1, 2, 3), (0, 1, 0) \}
   (b) \{ (1, 2, 2), (2, 1, 3), (0, 0, 0) \}
   (c) \{ (-2, 1, 3), (-1, 2, 3), (-1, -4, -3) \}

6. (10%) Compute the rank and nullity of the matrix
   \[
   \begin{bmatrix}
   1 & -2 & -1 \\
   2 & -1 & 3 \\
   7 & -8 & 3 \\
   5 & -7 & 0
   \end{bmatrix}
   \]
7. (10%) Find the eigenvalues and eigenvectors of the matrix \[
\begin{bmatrix}
2 & 2 & 3 \\
1 & 2 & 1 \\
2 & -2 & 1
\end{bmatrix}
\]

8. (20%) Solve the following differential equations.

(a) \[(2y - 6x)dx + (3x - 4x^2 y^{-1})dy = 0\]

(b) \[y'' - 6y' + 5y = 0\]

9. (10%) A financial consultant wishes to invest up to a total of $30,000 in two types of securities, one that yields 10% per year and another that yields 8% per year. Furthermore, she believes that the amount invested in the first security should be at most one-third of the amount invested in the second security. What investment program should the consultant pursue in order to maximum income?

(a) Please list the objective function and constraints of this problem.
(b) Solve this problem graphically.