1. Let $R$ be a binary relation on the set of all positive integers such that

   $a \ R \ b$ if and only if $a + b$ is an even positive integer


2. Draw one graph $G$ that satisfies as many of the following constraints as possible:
   
   (a) $G$ has 7 or fewer vertices.
   (b) $G$ has two connected components.
   (c) There is exactly 1 circuit (cycle) in $G$.
   (d) At least one vertex of $G$ has degree 3.
   (e) $G$ has no subgraph isomorphic to $K_3$, the complete graph of 3 vertices.

List clearly which of these constraints your graph satisfies. (15%)

3. The inorder and preorder traversals of a binary tree $T$ yield the following sequences of nodes, respectively.

   Inorder: $E A C K F H D B G$
   Preorder: $F A E K C D H G B$

Please draw the diagram of $T$. (10%)
4. Let $P(n)$ be the statement $2 \mid (2n-1)$.
   (a) Prove that $P(k) \rightarrow P(k+1)$
   (b) Show that $P(n)$ is not true for any integer $n$.
   (c) Do the results in parts (a) and (b) contradict the principle of mathematical induction?

5. Let $f$ be a function from $X$ to $Y$. Prove that $f$ is one-to-one if and only if $f(A \cap B) = f(A) \cap f(B)$ for all subsets of $A$ and $B$ of $X$.

6. Show that if any six numbers from 1 to 10 are chosen, then two of them will add up to 11.

7. We are given a red box, a blue box, and a green box. We are also given 10 red balls, 10 blue balls, and 10 green balls. Balls of the same color are considered identical. Determine the number of ways in which we can put the 30 balls into these 3 boxes so that:
   (a) No box contains any ball that has the same color as the box. (5%)
   (b) No constraint has to be satisfied. That is, every combination is permitted. (5%)

8. In each of the following groups of functions, choose the one that grows fastest asymptotically as $n \rightarrow \infty$.
   (a) Which function grows fastest?
   \[
   4n^2 \quad \frac{1}{2}n^3 - 6n \quad n^2 \log n \quad \frac{1}{2}n(n+1) \quad \frac{1}{n}
   \] (5%)
   (b) Which function grows fastest?
   \[
   n^3 \quad \sqrt[3]{n} \quad (\sqrt{n})^3 \quad \log n \quad (\sqrt[3]{n})^n
   \] (5%)
   (c) Which function grows fastest?
   \[
   \frac{n}{\log n} \quad n \log n \quad n^{\log n} \quad (\log n)^n \quad \log(n^3)
   \] (5%)