1. Find the solution of differential equation \( x'' - 4x = 3\delta(t) \) using Laplace transform and discuss initial values at \( t=0 \) as limits from the right (i.e. \( x(0^+), x'(0^+) \)). (25%)

2. There are various differential equations which can be reduced to Bessel's equation. To illustrate this, use the indicated substitution and find the general solution of the following equation in terms of Bessel function. (25%)
\[ x^2 y'' + xy' + (\lambda^2 x^2 - \nu^2) y = 0 \quad (\lambda x = z) \quad \nu = \text{constant} \]

3. Let \( f(x) = 1 + x \) for \( 0 \leq x \leq 1 \) (25%)
   (a) Expand \( f \) in a Fourier cosine series for a period of 2.
   (b) Expand \( f \) in a Fourier sine series for a period of 2.
   Hint: Half Range extension

4. A frictionless system of two masses coupled by two springs as shown in Fig. 1 is capable of oscillating as a whole in two different ways, called normal modes, with two different frequencies. (25%)
   (a) Obtain the governing differential equations of motion in matrix form. (Hint: Identify degrees of freedom first.)
   (b) Find the normal modes (eigenvectors) and their associated natural frequencies.

![Fig.1](image-url)