1. Can the permutations of \( \{1, 2, ..., n\} \) be arranged in a sequence so that adjacent permutations 
\[ p: p_1, ..., p_n \quad \text{and} \quad q: q_1, ..., q_n \]
(satisfy \( p_i \neq q_i \) for \( i = 1, ..., n \))
(a) Describe a graph model appropriate for solving the above problem. (10%)
(b) Solve the problem for \( n = 1, 2, 3 \). If there exist the answer, write the sequence; otherwise, give an explanation on your model. (6%)

2. Define the relation \( R \) on the set \( N \) of nonnegative integers as follows:
\[ (a, b) \in R \quad \text{iff the sum of the digits (decimal) of} \ a \ \text{equals the sum of the digits of} \ b. \]
(a) Show that \( R \) is an equivalence relation on \( N \). (10%)
(b) Let \( X = \{ x \mid x \in \text{equivalence class } [203], 0 < x < 100000 \} \), what is \( |X| \)? (9%)

3. Let \( f(1) = a, f(2) = b \), and \( f(n) = f(n-1) + f(n-2) \) for \( n > 2 \). Show that for all \( n \in \mathbb{N} \),
\[ \sum_{k=1}^{n} f(k) = f(n+2) - f(2). \] (10%)

4. Show that if \( 0 < a < b \) and \( 0 < c < d \), then \( a \cdot c < b \cdot d \). (10%)

5. Show that \[
\begin{pmatrix}
1 & 2 \\
3 & 6
\end{pmatrix}
\]
has no inverse. (10%)

6. Let \( n \) be a positive integer. Prove that
\[ \sum_{0 \leq k \leq n-1} \frac{1}{(n+k)(n+k+1)} = \frac{1}{2n} \] (15%)

7. Solve the recurrence relation
\[ s_n - 2s_{n-1} + s_{n-2} = 2 \]
with initial condition \( s_0 = 1 \) and \( s_1 = 1 \). (10%)

8. Find the coefficient of \( x^5z^4 \) in the expansion of \( (x + y + z + w)^9 \). (10%)