1. (15%) Determine where the following functions are increasing, decreasing, concave upward, and concave downward. And find the relative extrema and inflection points, and draw their graphs.

(i) \( f(x) = x^4 + 8x^3 + 18x^2 - 8 \)

(ii) \( f(x) = x^{2/3} \)

(iii) \( f(x) = \frac{x}{(x+1)^2} \)

2. (20%) Prove:

(i) \( \ln UV = \ln U + \ln V \)

(ii) The reflection of the point \((a, b)\) in the line \(y = x\) is \((b, a)\)

(iii) \( \lim_{n \to \infty} \left(1 + \frac{a}{n}\right)^n = e^a \)

(iv) \( \sum_{j=1}^{\infty} \frac{1}{j} = \infty \)

3. (20%) Compute the values of the given functions:

(i) \( \int \frac{x}{x+1} \, dx \)

(ii) \( \lim_{x \to 0} \frac{\sin^2 x}{\cos(3x) + 1} \)

(iii) \( \lim_{x \to 0} \frac{\sin(3x)}{1 + \sin(4x)} \)

(iv) \( \sum_{N=1}^{\infty} NP(1-P)^N \)

4. (15%) Suppose

\[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}; \quad -\infty < x < \infty \]

(i) Pove: \( \int_{-\infty}^{\infty} f(x) \, dx = 1 \)

(ii) Prove: \( \int_{-\infty}^{\infty} xf(x) \, dx = \mu \)

(iii) Prove: \( \int_{-\infty}^{\infty} (x - \mu)^2 f(x) \, dx = \sigma^2 \)
5. (10%) An environmental study of a certain suburban community suggests that the average daily level of carbon monoxide in the air will be

\[ c(p) = \sqrt{0.5p^2 + 17} \]

parts per million when the population is \( p \) thousand. It is estimated that \( t \) years from now, the population of the community will be \( p(t) = 3.1 + 0.1t^2 \) thousand. At what rate will the carbon monoxide level be changing with respect to time 3 years from now?

6. (10%) Suppose the output at a certain factory is \( Q = 2x^3 + x^2y + y^3 \) units, where \( x \) is the number of hours of skilled labor used and \( y \) the number of hours of unskilled labor. The current labor force consists of 30 hours of skilled labor and 20 hours of unskilled labor. Use calculus to estimate the change in unskilled labor \( y \) that should be made to offset a 1-hour increase in skilled labor \( x \) so that output will be maintained at its current level.

7. (10%) You are standing on the bank of a river that is 1 mile wide and want to get to a town on the opposite bank, 1 mile upstream. You plan to row on a straight line to some point \( P \) on the opposite bank and then walk the remaining distance along the bank. To what point \( P \) should you row in order to reach the town in the shortest possible time if you can row 4 miles per hour and walk 5 miles per hour?