1. For $n$ a positive integer, show that the following formula. (10%)
\[
\binom{2n}{n} + \binom{2n}{n-1} = \frac{1}{2} \binom{2n+2}{n+1}
\]

2. How many linear arrangements of the digits 0, 1, 2, ..., 9 either start with a 3 or end with a 7, or both? (10%)

3. Solve the following recurrence relation. (10%)
\[a_{n+1} - a_n = 2n + 3, \quad n \geq 0, \quad a_0 = 1.\]

4. Find the coefficient of $x^7$ in $f(x) = (1 + x + x^2 + x^3 + ..)^5$. (10%)

5. Let $G$ be an undirected (loop-free) graph with $v$ vertices and $e$ edges. How many edges are there in $\overline{G}$? Note that $\overline{G}$ is the complement of $G$. (10%)

6. In Fig. 1 shown below, answer the following questions.
   (a) Find the minimum spanning tree. (10%)
   (b) Find the shortest path from $a$ to $z$. (10%)

![Fig. 1](image-url)
7. Minimize the following expressions using a true table of map technique. (10%)
\[ f = ABC + \overline{A}BC + \overline{AB}C + \overline{ABC} \]

8. Find a well ordering of the set \{a, b, c, d, e, f, g\} that extends the relation \{(a, a), (a, g), (b, a), (c, a), (d, b), (d, c), (d, f), (e, d), (e, g), (e, f)\}. (10%)

9. A three-state finite state machine has \{0, 1\} as its input and output alphabets. Given the following input sequence and its corresponding output sequence, determine the machine. (10%)
Input sequence: 00010101
Output sequence: 011001110