1. (20%) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$, find $A^{10} = ?$

2. (10%) Use Gram-Schmidt process to find three orthonormal vectors from

$$s_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad s_2 = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \quad s_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

3. (10%) Let $T$ be the matrix transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ whose associated matrix is:

$$L = \begin{bmatrix} 1 & -2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$$

Let

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

and $V = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$ be ordered bases for $\mathbb{R}^2$ and $\mathbb{R}^3$, respectively. Determine the matrix representation of $T$ relative to $S$ and $V$.

4. (10%) Find the general solution of the equation

$$x^3 y'' + xy' - y = x \ln(x)$$
5. (20%) Apply the Laplace Transform

\[ F(s) = \int_0^\infty e^{-st} f(t) \, dt \]

(A) Find the transform of the equation \( f(t) = t \cdot \cos \omega t \).

(B) Using the transform, solve the Differential Equation

\[ y'' + y = 0 \]

with the initial \( y(0) = y'(0) = 1 \).

6. (10%) If \( f(x) \) and \( g(x) \) have period \( p \), show that \( h = af + bg \) (\( a, b \) constant) has the period \( p \).

7. (10%) Find the determinate of matrix \( A \), where

\[
A = \begin{bmatrix}
1 & 0 & 3 & 7 \\
4 & 2 & 0 & 1 \\
7 & 7 & 3 & 0 \\
5 & 0 & 6 & 8
\end{bmatrix}
\]

8. (10%) Please integrate the following function.

\[ \int z^k \, dz \]